Name: Solutions

Date: 3/18/2025

Math 130
Exam 2

Please show ALL your work on the problems below. No more than 1 point will be given to problems if you only provide the correct answer and insufficient work.

1. (24 points) Suppose you are going to make a bet with your friend on the result of drawing a single card from a standard poker deck. Specifically, you will win \$50 if you draw a red face card, you will lose \$10 if you draw any other red card, and you will lose \$5 is you draw any other card. Let *X* denote the amount of money you will win when playing this game once.

a) Find the probability distribution of *X*

x	$P(X=x)$
\$50	$\frac{6}{52}$
−\$10	$\frac{20}{52}$
−\$5	$\frac{26}{52}$

b) Find the expected value of *X*

$$\mu = \sum x\,P(X=x)$$

$$= (50)\left(\frac{6}{52}\right) + (-10)\left(\frac{20}{52}\right) + (-5)\left(\frac{26}{52}\right)$$

$$= \boxed{-\$0.58}$$

c) Find the standard deviation of *X*

$$\sigma = \sqrt{\left[\sum x^2 P(X=x)\right] - \mu^2} = \sqrt{(50)^2\left(\frac{6}{52}\right) + (-10)^2\left(\frac{20}{52}\right) + (-5)^2\left(\frac{26}{52}\right) - (-0.5769\ldots)^2}$$

$$= \boxed{\$18.41}$$

d) Explain the meaning of your answer from part (b)

If you make this bet many times, it's as if you lose about \$0.58 per bet.

2. (2, 3, 5, 6, 3 points) Stella the cook is good at burning the meals she prepares. In fact, the probability that she burns a meal is 28%. Assume that Stella burning a given meal is independent of the other times she burns a meal. Let X denote the number meals Stella burns among the next 21 meals she prepares.

a) What distribution does X have?

Binomial

b) Find the other 6 things you are supposed to list when solving problems for this kind of random variable.

$n = 21$

Success = Stella burns the given meal

Failure = Stella does not burn the given meal

$p = 0.28$

$q = 0.72$

X = The total number of meals Stella burns among the next 21 meals she prepares.

c) What is the probability that Stella burns exactly 7 meals?

$$P(X=7) = {}_{21}C_7 (0.28)^7 (0.72)^{21-7} = \boxed{0.1579}$$

d) What is the probability that Stella burns between 7 and 9 meals (inclusive)?

$$P(X=7 \text{ or } X=8 \text{ or } X=9) = P(X=7) + P(X=8) + P(X=9)$$

$$= {}_{21}C_7 (0.28)^7 (0.72)^{21-7} + {}_{21}C_8 (0.28)^8 (0.72)^{21-8} + {}_{21}C_9 (0.28)^9 (0.72)^{21-9}$$

$$= 0.1579 + 0.1074 + 0.0603 = \boxed{0.3256}$$

e) What is the expected value, standard deviation, and variance of X?

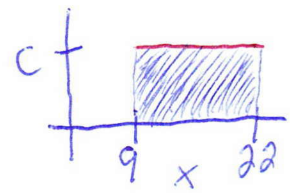
$$\mu = np = (21)(0.28) = \boxed{5.88}$$

$$\sigma = \sqrt{npq} = \sqrt{(21)(0.28)(0.72)} = \boxed{2.0576}$$

$$\sigma = npq = (21)(0.28)(0.72) = \boxed{4.2336}$$

3. (5, 3, 5 points) Suppose the random variable X has a uniform distribution on the interval [9, 22].

a) Find the value of c that makes this a probability distribution



Area under entire curve = 1

$b \cdot h = 1$

$13 \cdot c = 1$

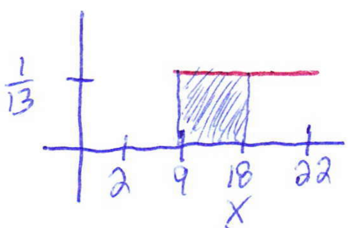
$\frac{13 \cdot c}{13} = \frac{1}{13}$

$\boxed{c = \frac{1}{13}}$

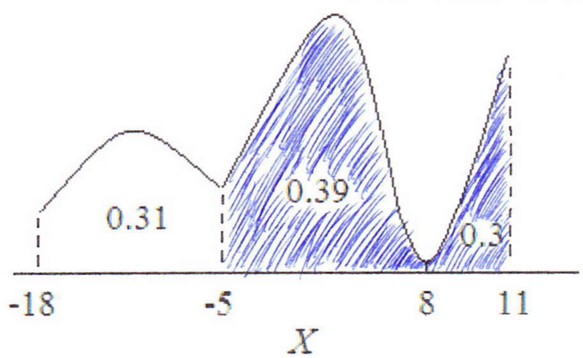
b) Find $P(X = 18)$

$= \boxed{0}$

c) Find $P(2 < X < 18)$ $= P(9 < X < 18) = b \cdot h = 9 \cdot \frac{1}{13} = \boxed{\frac{9}{13}}$



4. (3, 7 points) Suppose X is a random variable whose density curve is given below.



a) What are the possible values of X?

All decimals between -18 and 11

b) Find $P(-5 < X < 21)$

$= P(-5 < X < 11) = 0.39 + 0.3 = \boxed{0.69}$

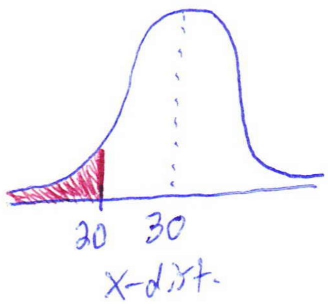
5. (28 points) The time it takes me to grade a stack of stats quizzes has a normal distribution with a mean of 30 minutes and a standard deviation of 6 minutes.

a) What is the probability that the next time I grade a stack of stats quizzes it will take me at most 20 minutes?

Let X be the time it takes to grade a randomly selected stack of quizzes.

X is normal

$\mu = 30 \quad \sigma = 6$

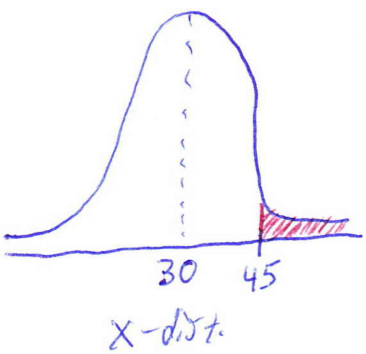


$P(X < 20) \overset{\text{Z-trans}}{=} P\left(\frac{X-\mu}{\sigma} < \frac{20-\mu}{\sigma}\right)$

$= P\left(Z < \frac{20-30}{6}\right)$

$= P(Z < -1.67) = \boxed{0.0475}$

b) What is the probability that the next time I grade a stack of stats quizzes it will take me more than 45 minutes?



$P(X > 45) \overset{\text{Z-trans}}{=} P\left(\frac{X-\mu}{\sigma} > \frac{45-\mu}{\sigma}\right) = P\left(Z > \frac{45-30}{6}\right)$

$= P(Z > 2.50) = 1 - P(Z < 2.50)$

$= 1 - 0.9938 = \boxed{0.0062}$

c) What is the probability that the next time I grade a stack of stats quizzes it will take me between 25 minutes and 32 minutes?

$P(25 < X < 32) \overset{\text{Z-trans}}{=} P\left(\frac{25-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{32-\mu}{\sigma}\right)$

$= P\left(\frac{25-30}{6} < Z < \frac{32-30}{6}\right) = P(-0.83 < Z < 0.33)$

$= P(Z < 0.33) - P(Z < -0.83)$

$= 0.6293 - 0.2033 = \boxed{0.4260}$

d) What does the probability you found in part (b) mean?

If I grade a stack of quizzes many times, it will take me more than 45 minutes to grade them about 0.62 % of the time.

6. (7, 10 points) A randomly selected lottery ticket has a 43% chance of being a winning ticket. Suppose you go to the store and buy 200 lottery tickets. Let X denote the total number of tickets you purchased that are winning tickets. Assuming that a ticket being a winning ticket is independent of if any other ticket is a winning ticket, use the normal approximation to the binomial distribution to answer the following questions:

a) What is the probability that less than 80 of your tickets will be winning tickets?

$\boxed{\text{Binomial}}$

$n = 200$

Success = if the given lottery ticket is a winner

Failure = if the given ticket is not a winning ticket

$p = 0.43$

$q = 0.57$

X = The total number of winning tickets among the 200 purchased tickets

Condition: $npq \geq 10$?

$(200)(0.43)(0.57) = 49.02 \geq 10$ ✓

So X is approximately normal.

$\mu = np = (200)(0.43) = 86$

$\sigma = \sqrt{npq} = \sqrt{(200)(0.43)(0.57)}$

$= \sqrt{49.02}$

Binomial

$P(X < 80) = P(X \leq 79)$

$\overset{\text{C.C.}}{=} \underset{\text{Normal}}{P(X \leq 79.5)}$

$\overset{\text{Z-trans}}{=} P\left(\frac{X-\mu}{\sigma} \leq \frac{79.5-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{79.5-86}{\sqrt{49.02}}\right)$

$= P(Z \leq -0.93)$

$= \boxed{0.1762}$

b) What is the probability that between 85 and 100 of your tickets will be winning tickets (excluding 85 and including 100)?

$\underset{\text{Binomial}}{P(85 < X \leq 100)} = P(86 \leq X \leq 100) \overset{\text{C.C.}}{=} P(85.5 \leq X \leq 100.5)$

$\overset{\text{Z-trans.}}{=} P\left(\frac{85.5-86}{\sqrt{49.02}} \leq \frac{X-\mu}{\sigma} \leq \frac{100.5-86}{\sqrt{49.02}}\right)$

$= P(-0.07 \leq Z \leq 2.07)$

$= P(Z \leq 2.07) - P(Z \leq -0.07)$

$= 0.9808 - 0.4721$

$= \boxed{0.5087}$

7. (3, 7 points) Consider the experiment where in order to complete the experiment once you have to first flip a single coin then roll a single die.

a) What is the sample space?

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

b) Define a random variable on this experiment.

Here are 3 examples ...

X = The number the die landed on (meaning H1 → 1, H5 → 5, T3 → 3 and so on)

$$Y = \begin{cases} 1 & \text{if the coin landed on heads} \\ 2 & \text{if the coin landed on tails} \end{cases}$$

$$Z = \begin{cases} 1 & \text{if coin landed on heads and die landed on an even number} \\ 5 & \text{if coin landed on heads and die landed on an odd number} \\ 7 & \text{if coin landed on tails and die landed on an even number} \\ -3 & \text{if coin landed on tails and die landed on an odd number} \end{cases}$$

Some formulas you may need:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad P(A \cup B) = P(A) + P(B)$$

$$P(A \cap B) = P(A) \cdot P(B \mid A) \qquad P(A \cap B) = P(A) \cdot P(B)$$

$$P(\overline{A}) = 1 - P(A) \qquad P(at \quad least \quad one) = 1 - P(none)$$

$$EV = \mu = \sum xp(X = x)$$

$$Var = \left[\sum x^2 p(X = x)\right] - \mu^2$$

$$\sigma = \sqrt{\left[\sum x^2 p(X = x)\right] - \mu^2}$$

$$P(X = x) = {}_nC_x p^x q^{n-x} \qquad \mu = np \qquad \sigma^2 = npq \qquad \sigma = \sqrt{npq}$$

$$Z = \frac{X - \mu}{\sigma}$$